

Empirical and mathematical approaches to rockfall protection and their practical applications

Approches empirique et mathématique de la protection contre les éboulements et applications pratiques

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SUMMARY: The paper first describes the state of the art in rockfall evaluation and in the design of protective measures. Subsequently a computer program is presented which calculates the velocity and kinetic energy of spherical rockfall along its path. Free falling, rolling, sliding and trajectories and their transitions are considered as well as energy dissipation during impact, friction between rockfall and slope surface and the angular velocity of the rockfall. Protective structures can be introduced at all points of the slope profile. Appropriate safety concepts are presented. Finally the possibilities and shortcomings of analytical procedures in general, and of the presented solution in particular are discussed.

RESUME: Avant de présenter un programme pour l'ordinateur qui calcule la vitesse ainsi que l'énergie cinétique de chutes de pierres d'une forme sphérique l'état actuel de la connaissance dans l'évaluation des risques par chutes de pierres est donné. Le programme prend en considération la chute directe, le roulement, le glissement et les trajectoires ainsi que leurs transitions et décrit la dissipation d'énergie pendant l'impact, la friction entre la pierre et la surface du talus ainsi que sa vitesse angulaire résultante. Des parades peuvent être introduit à l'endroit quelconque. Une conception valable de sécurité est donnée. Les possibilités et limites de solutions analytiques en général et par rapport au procédé de calcul présenté sont discutés.

ZUSAMMENFASSUNG: Zunächst wird der Stand der Kenntnisse bei der Abschätzung von Steinschlagrisiken und beim Entwurf von aktiven Sicherungsmaßnahmen dargestellt. Anschließend wird ein Rechenprogramm vorgestellt, das sowohl die Geschwindigkeit als auch die kinetische Energie kugelförmiger Steinschläge entlang deren Bahn berechnet. Dabei werden sowohl der Freie Fall, als auch Rollen, Gleiten und der Schiefe Wurf samt ihrer Übergänge berücksichtigt. Das verwendete Rechenverfahren beschreibt auch die Energieverluste beim Stoß, sowie die Reibung zwischen Steinschlag und Hangoberfläche, und die daraus resultierende Winkelgeschwindigkeit des Steinschlags. Schutzbauwerke können an beliebiger Stelle in die Berechnung eingefügt werden. Im Zusammenhang mit der Berechnung werden aussagefähige Sicherheitskonzepte aufgestellt. Schließlich wird auf die Möglichkeiten und Grenzen analytischer Lösungen generell und im Bezug auf das vorgestellte Rechenverfahren eingegangen.

INTRODUCTION

During the past 30 years considerable progress was made in assessing the overall stability of rock slopes. Several new failure modes like toppling and buckling have been detected and powerful design tools like the finite element method have been developed. However, the local stability of the slope face did not find a similar attention. The reason might be that rockfall, as a result of insufficient stability of steep rock faces, is less spectacular than large slides which could interrupt highways and railway lines for weeks.

Nevertheless a lot of money has to be spent every year to maintain or improve rockfall protective measures in the alpine countries. As is generally admitted this money has to be spent on an empirical base, because our theoretical knowledge about the mechanics of rockfall is limited up to now.

Due to an increasing public concern on the safety of technical structures, this situation now changes toward a more rational and reproducible design. But still, no

commonly accepted definitions of basic terms exist. For example most authors give definitions of rockfall, referring to the volume, which is involved in one single event. JOHN & SPANG (1979) give an upper limit of 0.1m³/event which can be handled by ordinary catch structures. ROCHET (1987) gives a limit of about 250 m³/event. Regarding the design of protective structures, it seems to be more practical to use kinetic energy as a criterium to classify rockfalls. According to SPANG (1987), rockfall therefore is limited to a kinetic energy of about 500 KNm. Larger events require active measures, which prevent loose rocks from coming into motion. Rockslides, avalanches, mass flows and other larger movements are not included in the following considerations. As for their concern, it is referred to SCHEIDEGGER (1973), KÖRNER (1976) and REIK & HESSELMANN (1977).

STATE OF THE ART

One of the first, describing the phenomenon from a geological point of view, was HEIM (1882). But the first practicable approach

to the design of rockfall protective measures was not published until 1963. The author, RITCHIE (1963), reported on extensive field experiments, using artificially triggered small scaled rockfall to test the effectiveness of ditches and fences. Using his design rules, it became the first time possible to select appropriate depths and widths of ditches, also in combination with fences or guard rails, if necessary, according to the actual slope inclination.

Large scaled rockfall was analysed by BRÖLLI (1974). He described some fundamental observations on the paths and behaviour of boulders, including relations between volume, height of rebound and width of trajectories following free falling of several hundred meters. He also pointed out rock break-ups along the paths, which reduce the kinetic energy of the individual particles at a certain line of interest.

LIED (1977), from experience, gave a criterium to assess the maximum reach of rockfalls, such contributing to risk evaluation. He also described some kinds of protective measures, like concrete walls, earth and rockfills and gabions. The dimensions of these structures had to be selected by experience. Because of the better access, protective measures mostly were erected at the toe of the slopes.

JOHN & SPANG (1979) gave a systematic review on the conditions, under which rockfall occurs, also based on field tests. They described different active and passive protective measures and presented criteria for their selection.

The above described investigations proved helpful in cases of uniform or bi-linear concave slope inclinations. But with more complex morphologies, the transfer of test results from one slope to another remained delicate. Field tests generally are costly and in some cases artificially triggered rockfall led to the same risk, as naturally generated one would have done. Obviously there was a lack of an universally applicable design method, based on mathematical models and able to perform parametric studies at low costs.

The first step toward an analytical solution was also done by RITCHIE (1963). An algorithm was proposed to calculate the velocity and path of a rockfall, starting with free falling and getting into trajectory after impact on the slope surface. The approach was based on elementary laws of motion. The model comprised one impact only and the relation between the velocities before and after this impact. No factor of restitution was considered. Friction between rockfall and slope surface - resp. the influence of the angular momentum and velocity is discussed only in a qualitative manner.

A similar approach was suggested by ESCHENBACH & KLENGEL (1975). Now a factor of restitution was defined, describing the energy dissipation during impact. The relevant value was found by laboratory tests, using rock spheres. The angle of rebound was pre-selected, as to give the maximum width/height of trajectory for the design of catch structures, located at the

toe of the slope. No friction between rockfall and slope surface was considered.

The first analytical approach, using a computer, was announced by PITEAU & CLAYTON (1977). Their model used a factor of restitution and a rough slope surface to enable rolling blocks, but apparently no angular momentum is considered during collision. The rockfall starts by free falling too, getting into trajectory after collision with ground surface. A complete sequence of trajectories and impacts is calculated, until the rockfall reaches a pre-selected line, for example a catch fence or a road. The slope geometry is represented by 2-dimensional straight segments. These segments are divided into 2 cells. The angle between the cells can be varied about the mean angle to test the influence of macroscopic roughnesses. By use of histograms, showing frequency vs. distance from toe, probabilistic studies can be included in risk evaluation. Hypothetical catch walls could be combined with variable slope geometries to evaluate their effectiveness. The model too, was based on elementary laws of motion. To calculate the angular relations between the velocity vector before and after the collision, SNELL's law of refraction from optics was used.

Meantime, other authors have published mathematical approaches to rockfall control too. Amongst them are AZIMI et al. (1982), BOZZOLO and PAMINI (1982), and FALCETTA (1985). The state of the art, which had been reached until 1987, was represented by 3 publications, presented at the 6th International Congress on Rock Mechanics in Montreal.

DESCOEUDRES & ZIMMERMANN (1987) presented a three-dimensional computer model, using fundamental equations of the dynamics of rigid solids. The model describes the spatial path and the velocities of a single, rigid boulder of prismatic or ellipsoidal shape, starting from a pre-selected point above a slope by free-falling.

The slope surface, which is either inelastic or plastic, is discretized into quadrilateral elements. Collisions are governed by coefficients of restitution and friction, the former being either constant or depending on the actual conditions of each impact. Angular momenta resp. angular velocities and their changes during impact are considered. Along the path of the rockfall, rolling and sliding can occur.

The model was used for risk evaluation, especially for the determination of the maximum reach of rockfalls without regarding catch structures.

From an example, some general conclusions can be drawn.

- Bigger blocks ($5-10 \text{ m}^3$) show straighter paths, their points of arrest lie higher.
- Decrease in the coefficient of friction causes an increase in velocities and in lengths of rebounds.
- Decrease in the coefficient of restitution causes a decrease in velocities and earlier arrest.

Because of the difficulties in assessing the input data, governing the path of rockfalls, ROCHET (1987) suggested two basic approaches to risk evaluation and to the study of protective measures.

- Calculation of envelope trajectories, aiming at the analysis of the limit conditions of rockfall propagation, as for maximum heights, velocity, reach and kinetic energy.

- Calculation with random variables, aiming at analysing dispersion, resulting in randomly distributed paths within the maximum reach.

The paper also referred to two- and three-dimensional computer programs, which were reported to be able to execute the above described analyses, but no details on the applied algorithms were given.

Another mathematical model was presented by SPANG (1987). Since then, the model was essentially improved and additional practical experience was gained. The actual state of this so-called Rockfall Computer Programm (RCP) is reported below.

ROCKFALL COMPUTER PROGRAM

As for overall stability, rock slope design is focused on the primary mode of rupture, such as sliding, buckling toppling or other. The aim is to prevent rock masses from getting into motion. In the case of rockfall, the design deals with rock masses, which are already in motion. The failure modes therefore are replaced by modes of motion. 4 elementary modes of motion can be distinguished:

- free falling
- bouncing
- rolling
- sliding.

Transitions exist between these modes of motion, rolling and sliding even can occur simultaneously. The transition can happen almost continuous, or abrupt, by impact. Consequently the Rockfall Computer Program selects the primary mode of motion amongst the above mentioned possibilities, when the coordinates of the starting point are given. If the primary mode is pre-selected, the program checks, if the chosen initial movement is possible or not. The subsequent calculation of the rockfall path is based on elementary laws of motion of rigid bodies, including NEWTON's Laws and his theory of collision.

The calculation is executed iteratively, according to the chosen time interval. After each time step, the program checks, if the calculated path resp. the actual position of the rockfall is kinematically possible or not. This examination includes the question, if the actual kind of motion can be continued or not. Depending on the result of this check the actual kind of motion is continued for the next step, or the transition to another mode of motion is calculated. If the tangential velocity becomes smaller than the critical value, the calculation is finished.

The same thing happens, if the rockfall leaves the given geometry. If the rockfall hits an obstacle, and a further movement toward the toe of the slope is kinematically impossible, the coordinates of the point of impact, the direction of the impact and the kinetic energy at the moment of impact is given and the calculation is finished too. In any case, where the checking shows a position, which is not possible, the correct time for the last possible position on that path is calculated, before a transition to another mode of motion, an impact and so on is regarded.

The following transitions are kinematically possible and are regarded:

- rolling/sliding and sliding/rolling;
- free falling/trajectory
- trajectory/rolling/sliding
- trajectory/trajectory

Until now, the geometry of the rockfall is idealized and restricted to spheres.

Considering friction between the rockfall and the slope surface, a planar, excentric impact is regarded. The resulting normal component of the velocity is governed by the factor of restitution, according to NEWTON's theory of particle collision. The tangential component relates to the tangential translational velocity and to the angular velocity of the rockfall. Its change during impact depends on the duration and relation of sliding and sticking phases during impact. Slope geometry is discretized into 2-dimensional slices with vertical boundaries and straight surfaces. The width and the location of boundaries can be chosen according to the local requirements. For input and output a global cartesian frame of reference is used.

According to the use of planar, 2-dimensional slope sections, a planar velocity field is assumed. The profiles have to be selected parallel to the dip direction of the slope surface.

The program was written in FORTRAN 77. A keyboard is used for input. The output can be shown on a display, or plotted resp. printed, using a commercial plott-program.

The Rockfall Computer Program consists of one main program, 22 subroutines and 3 data-files. The subroutines provide the calculation of slope and path geometries, different start and different subsequent modes of motion, transitions from one mode of motion into another and of the impact on protective structures or obstacles. The data-files are used to store input data as well as provisional results for test purposes, tracing and speeding up parametric studies, for documentation and output. A flow chart is shown in figure 1.

The hardware consists of an IBM AT 02 unit, with a 20 MB hard-disk and 2 floppy-disks 360 KB/1 2 MB. Results can be shown on an IBM colour display, on an Epson FX 100 + printer, and on a Hewlett Packard 7345 A plotter. To calculate the path of a rockfall with 20 impacts takes less than two seconds.

The required input data are already described in detail by SPANG (1987). Only

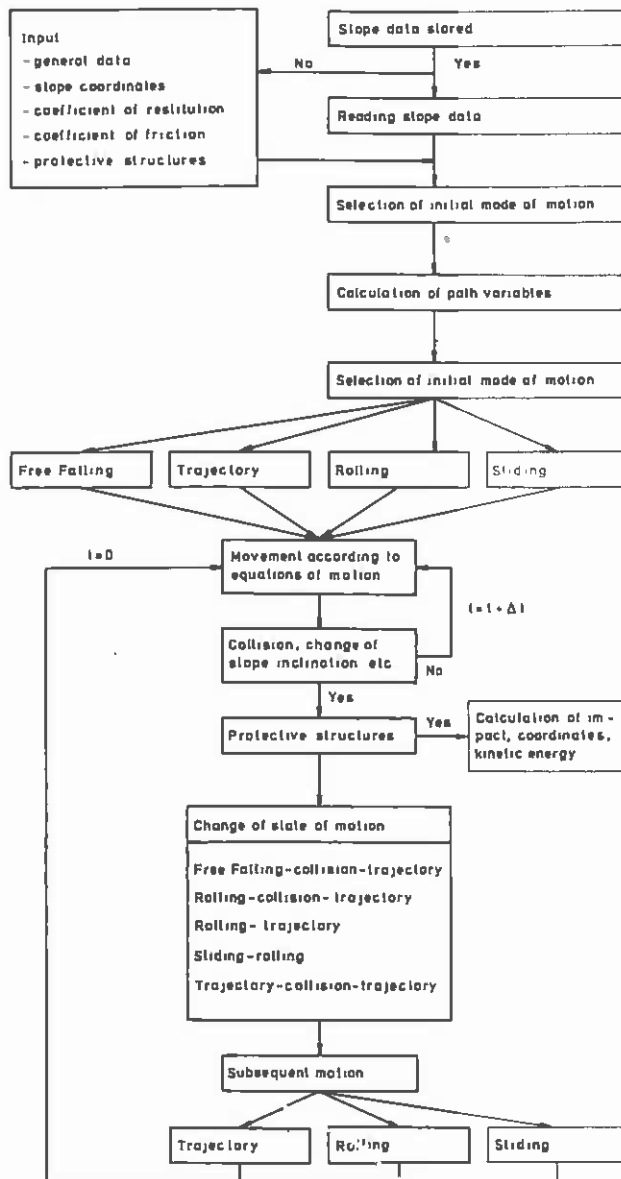


Figure 1: Flow chart of the Rockfall Computer model

two additional aspects will be discussed below, the coefficient of restitution and the factor of safety.

COEFFICIENT OF RESTITUTION

The coefficient of restitution is one main factor influencing the path of rockfalls. It expresses the amount of energy dissipation during impact. It is obvious that pure elastic behaviour cannot be expected. Only in few cases- for example in a swamp- nearly plastic ground behaviour will occur. In most cases, the ground will behave elastoplastic, with a considerable amount of dissipated energy. Because no systematic determination of the coefficient of restitution is available until now, experiences from other fields have to be used. From the dynamic compaction method, where also impacts play an important role, SMOLTZYK (1983) describes the following energy- transforming phenomena

- generation of elastic waves
- soil displacements under constant volume
- compaction (reduction of volume by displacements of grains)
- destruction of grains
- air resistance, rolling and sliding resistance.

The effect of all these phenomena is summarized in the coefficient of restitution. According to NEWTON's theory of particle collision (see for example SZABO (1966)), this coefficient is defined as (1)

$$e = \frac{v_2 - v_1'}{v_1 - v_2}, \quad 1 = e = 0$$

where v_1 represents the particle velocity before, v_1' after the collision of two particles 1 and 2. According to SZABO (1966) the coefficient is by no means a material constant, but depends on particle velocities, particle geometries and materials. It is not possible to deduct it from theoretical considerations and has to be selected therefore either by tests or by experience. For free falling tests equation (1) can be written as

$$e = \sqrt{\frac{h'}{h_0}} \quad (2),$$

where h_0 represents the original height of release, h' the rebound height after impact. Adequate rebound tests were conducted for example by CAMONUOVO (1977).

Because of the dependence of the coefficient of restitution on geometries and velocities and because of the lack of model laws, real boulders should be used with realistic original heights of release. These requirements usually can be fulfilled in the case of small scaled rockfall from limited heights only.

The heights of rebound after the first impact can be directly taken from video films, high-speed camera takes or from back analysis, using a geophone resp. a seismograph for registration of the flight time between the first and the second impact.

There are very few publications on the order of magnitude of the coefficient of restitution. HABIB (1977) reported e-values between 0.5 and 0.78. DESCOEUDRES & ZIMMERMANN (1987) gave values between 0.4 and 0.85. None of them gave a detailed description of the ground, for which these values should be representative. According to the author's field experience and with relation to SPANG (1987) these values seem relatively high and may be valid only for sound rock. For overburden and debris, values below 0.1 seem appropriate. As already mentioned there is a lack of systematic investigations. Obviously a wide field for further experiments exists.

SAFETY CONCEPTS

By using the presented Rockfall Computer Program it is quite simple to introduce meaningful safety factors. Of all kinds of barriers, fences, guardrails, walls, and so on, the first condition to be fulfilled is



Fig. 2: Brocken tie within a rail and tie wall near Kirchheim/Teck FRG.

that of a sufficient height to catch all probable rockfalls. The critical height is found by a variation of the input data within the probable range and the calculation of an envelope trajectory, as suggested by ROCHEI (1987). The quotient of the chosen height and this critical value define the first safety factor. The factor itself may be selected according to the uncertainties and risks of each situation. In normal cases values between two and three seem reasonable.

Additionally the structure itself may be subjected to usual safety concepts in structural engineering, based on the maximum kinetic energy of rockfall. This value can be taken from parametric studies too. Fig. 2 shows a typical example of an undersized rail and tie wall.

A similar safety factor can be applied to the width of berms, if it is defined as the quotient of the width of the berm and the maximum reach of rockfall on this berm. Values larger than 1 normally are uneconomic. If a higher factor of safety should be required, it may be cheaper to add a fence at the outer corner or install rows of boulders. Analogous considerations lead to comparable safety concepts for ditches, gabions, earth fills and other protective measures.

POSSIBILITIES AND LIMITATIONS OF ANALYTICAL MODELS

The above described Rockfall Computer Program was developed to fulfill the following tasks:

- Risk evaluation, i.e. answering the question, if an object within or below a slope is endangered by rockfall or not.
- Efficiency control, i.e. checking if existing or planned protective structures are able to fulfill the requirements.
- Optimization of the position and the height of planned protective structures.

- Delivery of input data for the structural design and the dimensioning of protective structures.
- Execution of parametric studies, as to overcome the uncertainties of input data by a sufficient number of tests with variable input data, including the effects of minor morphological details of the slope's surface.

By use of the Rockfall Computer Program

- Design becomes rational and reproducible.
- Alternative solutions can be easily compared. (for example ditch vs. fence; one heavy fence at the toe or several smaller ones within the slope).
- Design loads for structural analysis can be determined realistically.
- Safety factors can be easily introduced as for height and for kinetic energies.

Compared to SPANG (1987) the following improvements exist.

- Rolling as an initial or subsequent mode of motion is now correctly modelled.
- Changes of the angular velocity during impact are considered in a mechanically appropriate way.
- Transitions from one mode of motion to another are now independent from slice boundaries.
- Sliding and rolling at the same time are considered now.

There are, of course, some limitations of the suggested procedure. Some of them are of a general nature and are common for all analytical rockfall models.

As stressed by many practitioners, there usually are considerable uncertainties as for the input data. To the experience of this author, the problem can be satisfactorily solved by the execution of parametric studies and the use of resulting envelope trajectories in rockfall design. Besides, the possibilities of parametric studies exceed the potential of rockfall tests by far.

The effect of the unavoidable simplification of the slope's geometry can be eliminated to a certain degree by using very small slices. This may approach the real curved lines to an acceptable degree. By the way, surveying also is based on linear connections between neighbouring measurements. Of course the analytical model cannot lead to a higher accuracy as that of the field data, on which it is based. Up to now, all analytical solutions are restricted to one boulder at one time and at one place. No interaction between several boulders is considered. It is not completely excluded that the kinetic energy of a certain boulder at a certain point may be higher after the collision with another boulder than it was before. This uncertainty can be eliminated by the choice of higher safety factors in structural design of protective measures.

None of the known analytical models considers rock break-up's along the path of a rockfall. As a matter of fact, however, these rock break-up's are quite common, as shown by the observations of BROILI (1974) and by the laboratory tests of CAMONUOVO (1977). The reason for the break-up's is shown by acceleration measurements, executed on free falling weights, used in dynamic compaction of soils. So BEINE (1986) revealed accelerations up to about 100 g during impact. If the resulting reduction of the size of boulders cannot be foreseen from geological reasons, rock break-up's will lead to an adequate reduction of the kinetic energy of the generated particles. Thus the design will be on the safe side.

As already stressed the suggested procedure is restricted to 2-dimensional situations. Thus not only the mathematical approach becomes easier and the runtime shorter, but the requirements for surveying became economically feasible also in usual rockfall problems. The resulting limitation is considered to be neglectable in most cases, because the lateral movement of the rockfall is either limited by morphological features of the slope or is not decisive, because of the comparably excessive lateral extension of the slope under consideration. Because of geometrical uncertainties, also by use of 3-dimensional approaches a considerable lateral safety zone on both sides of the calculated paths will be required. As for the kinetic energies, the maximum values obviously will be reached along profiles parallel to the dip direction of the slope's surface.

Until now, the described Rockfall Computer Program is restricted to spherical rockfall geometries. From mechanical reasons, spheres represent the most unfavorable case as for reach and kinetic energies. Thus the results of calculations using spheres will be on the safe side. Besides, the effect of other geometries seems to be not decisive, following RITCHIE (1963).

OUTLOOK

On the base of the actually available methods and experiences, a rational risk evaluation and an economic design of protective measures became feasible. Shortcomings of the analytical methods can be compensated by parametric studies. However, for further development in rockfall evaluation and protection research should concentrate on the coefficient of restitution and its various dependencies as well as on a classification or standardization of catch structures according to their ability to sustain impulses of various magnitudes. Until systematic investigations on these subjects are available, rockfall tests by artificially triggered rockfall will be not dispensable in complex situations.

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